

(Verdad de A)

Tautología — $V(A) = 1$

Contradicción — $V(A) = 0$

Contingencia — $V(A) = 1 \text{ y } 0$

1°							2°				1°							3°				1°			
$(p \rightarrow q)$							$\wedge$				$(q \rightarrow p)$							$\rightarrow$				$(p \leftrightarrow q)$			
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$\rightarrow$ tautología

$$(a) \left. \begin{array}{l} p \rightarrow q \\ q \rightarrow p \end{array} \right\} (p \rightarrow q) \xrightarrow{2^o} (q \rightarrow p) : A$$

1	1	1	1	1	1	1
1	0	0	1	0	1	1
0	1	1	0	1	0	0
0	1	0	1	0	1	0

↳ Contradição

Contradição: $I(p) = 0$
 $I(q) = 1$

$$\begin{aligned} I(p \rightarrow q) &= 1 \\ I(q \rightarrow p) &= 0 \end{aligned}$$

$$[(p \vee \neg q) \wedge (r \rightarrow \neg p)] \rightarrow (\neg r \rightarrow q)$$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
	1	1	0	1	1	0	1	0	1	0	1	0	1	0	1
	1	1	1	0	0	1	0	0	1	0	1	0	1	0	1
→	1	1	1	0	1	0	1	0	1	0	1	0	1	0	1
	0	0	0	1	0	1	1	1	0	1	0	1	0	1	0
	0	0	0	1	0	1	1	1	0	1	0	1	0	1	0
	0	1	1	0	1	1	1	0	1	0	1	0	1	0	1
	0	1	1	0	1	1	1	0	1	0	1	0	1	0	1

$I(p) = 0$ $I(q) = 0$ $I(r) = 0$ $\rightarrow$ Contingencia

$$\models \not\models p \wedge \neg p$$

$$\{p_1, p_2, \dots, p_n\} \models C$$

$$(p_1 \wedge p_2 \wedge \dots \wedge p_n) \rightarrow C$$

$$(a) (p \vee q) \rightarrow p$$

$$\text{1. } \neg((p \vee q) \rightarrow p)$$

$$\text{2. } p \vee q$$

$$\text{3. } \neg p$$

$\alpha 1$

$$\begin{array}{cc} \text{4.1 } p & \text{4.2 } q \end{array}$$

(B2)

$$I = \vee$$

$$I(p) = 0$$

$$I(q) = 1$$

contingencia

$$\frac{P_1}{P_2} \quad C$$

$$\{P_1, P_2\} \vdash C$$



$$\begin{array}{l} \text{1. } P_1 \\ \text{2. } P_2 \\ \text{3. } \neg C \\ \vdots \end{array}$$

$$(P_1 \wedge P_2) \rightarrow C$$



$$\neg((P_1 \wedge P_2) \rightarrow C)$$

$\alpha - A \wedge B$	$\beta - A \vee B$
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$$\neg(A \vee B)$$

$$\neg(A \rightarrow B)$$

$\vdots$

(b) $(q \wedge r) \rightarrow s, (r \rightarrow s) \rightarrow t \models (q \vee s) \rightarrow t$

~~1.~~ $(q \wedge r) \rightarrow s$

~~2.~~ $(r \rightarrow s) \rightarrow t$

$\Rightarrow$ ~~3.~~ $\neg[(q \vee s) \rightarrow t]$

4. $q \vee s$

5. $\neg t$

$\} (\alpha 3)$

~~6.1.~~ $\neg(r \rightarrow s)$ 6.2 t_x ($\beta 2$)

7. r 8. $\neg s$ $\} (\alpha 6.1)$

~~9.1.~~ $\neg(q \wedge r)$ 9.2 s_x

10.1 q 10.2 s ($\beta 4$)

11.1 $\neg q_x$ 11.2 $\neg r_x$ ($\beta 9.1$)

($\beta 1$)

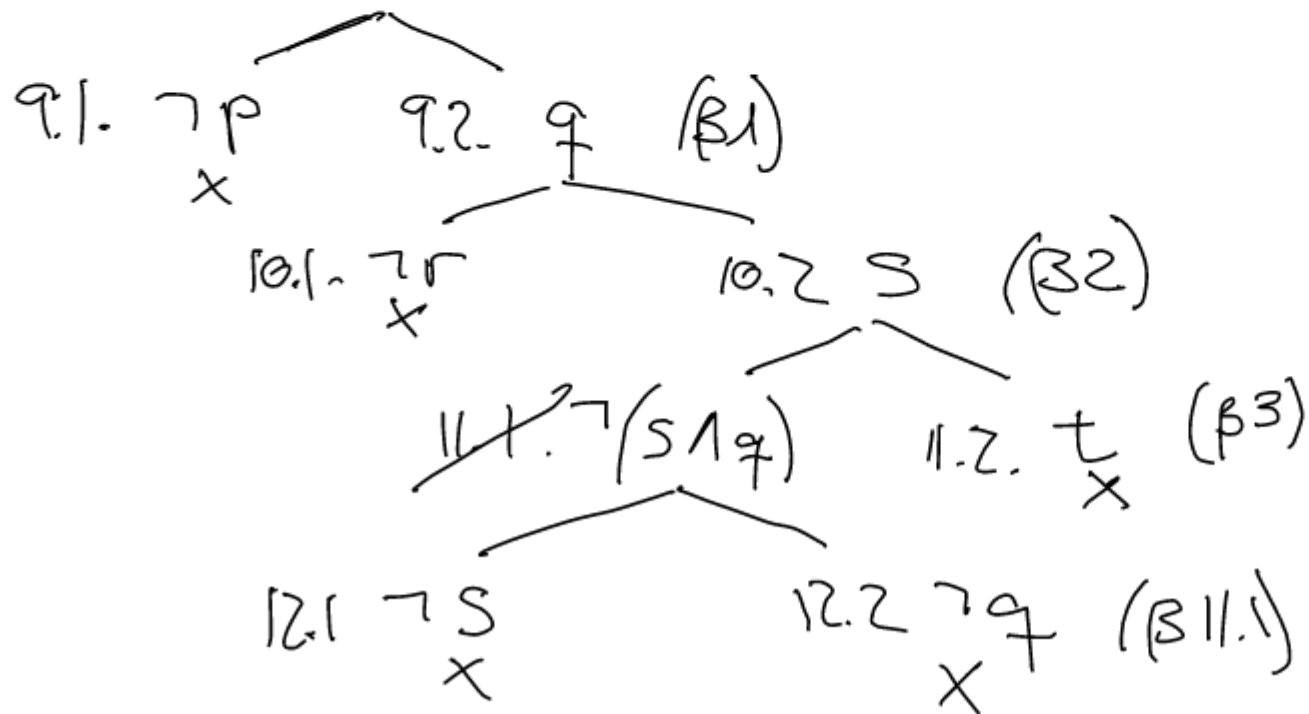
tautología
cierran todas
las ramas

α $A \wedge B$	β $A \vee B$
$\neg(A \rightarrow B)$	$A \rightarrow B$
	$\neg(A \wedge B)$

~~1.~~ $p \rightarrow q$
~~2.~~ $r \rightarrow s$
~~3.~~ $(s \wedge q) \rightarrow t$
~~4.~~ $\neg[(p \wedge r) \rightarrow t]$
 5. $p \wedge r$
 6. $\neg t$ } ($\alpha 4$)
 7. p
 8. r } ($\alpha 5$)

(β) $A \rightarrow B$

$\neg A$ B



(DN)

$\{\neg, \wedge\}$ définir resto ...

$$A \rightarrow B \equiv \neg A \vee B \equiv \neg(A \wedge \neg B)$$

$$\Rightarrow A \wedge B \equiv \neg(\neg A \vee \neg B) \equiv \neg(A \rightarrow \neg B)$$

$$\Rightarrow A \vee B \equiv \neg(\neg A \wedge \neg B) \equiv \neg A \rightarrow B$$

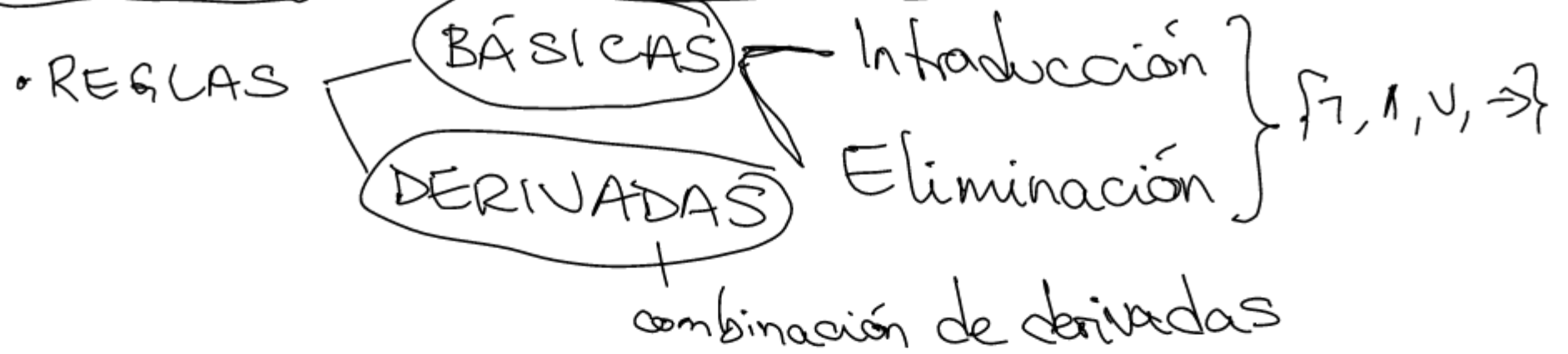
$$\frac{\alpha(A \wedge B)}{\neg(A \rightarrow B)}$$

$$\frac{\beta(A \vee B)}{A \rightarrow B}$$

✱

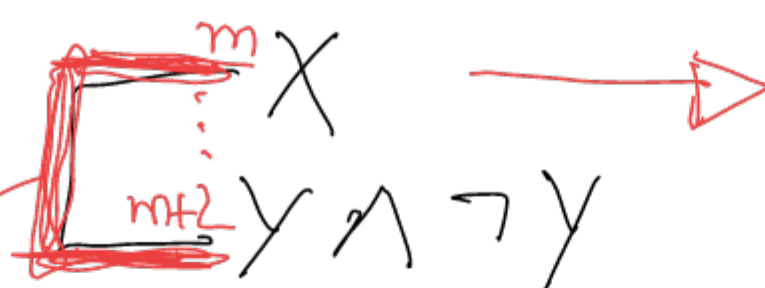
✱

DEDUCCIÓN NATURAL - Sintáctica

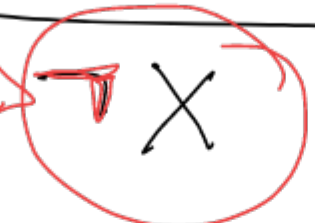


• ESTRATEGIAS

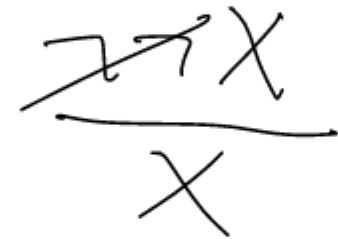
$R1 \rightarrow$



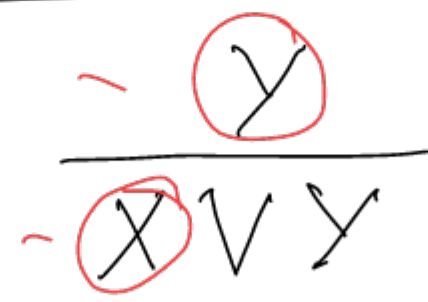
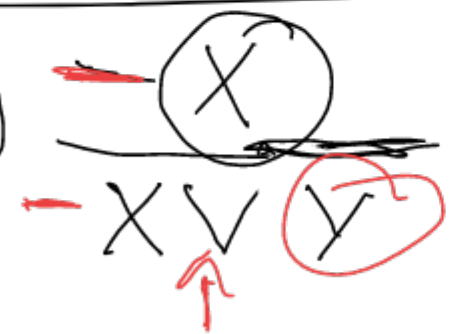
Supuesto



$RE \neg$



$RI \vee$



$= p \vee q$



REV

$$\begin{array}{r}
 \begin{array}{c}
 \boxed{\begin{array}{c} x \\ \vdots \\ z \end{array}} \\
 \boxed{\begin{array}{c} x \\ \vdots \\ z \end{array}}
 \end{array}
 \end{array}
 \begin{array}{c}
 x \vee y \\
 \vdots \\
 z
 \end{array}$$

$$z$$

$$\begin{array}{r}
 \begin{array}{c}
 \boxed{\begin{array}{c} p \\ \vdots \\ q \end{array}} \\
 \boxed{\begin{array}{c} p \\ \vdots \\ q \end{array}}
 \end{array}
 \end{array}
 \begin{array}{c}
 p \vee q \\
 \vdots \\
 r
 \end{array}$$

$$r$$

RIA

$$\frac{x}{y} \quad \frac{y}{x}$$
$$\frac{\quad}{x \wedge y} \quad \frac{\quad}{x \wedge y}$$

$$\begin{array}{c} \vdots p \\ \vdots \\ q \\ \hline p \wedge q \end{array}$$

REA

$$\frac{-x \wedge y}{x} \quad \frac{x \wedge y}{y}$$

$$\begin{array}{l} 5 p \wedge q \\ - p \text{ RE } 5 \end{array}$$

→

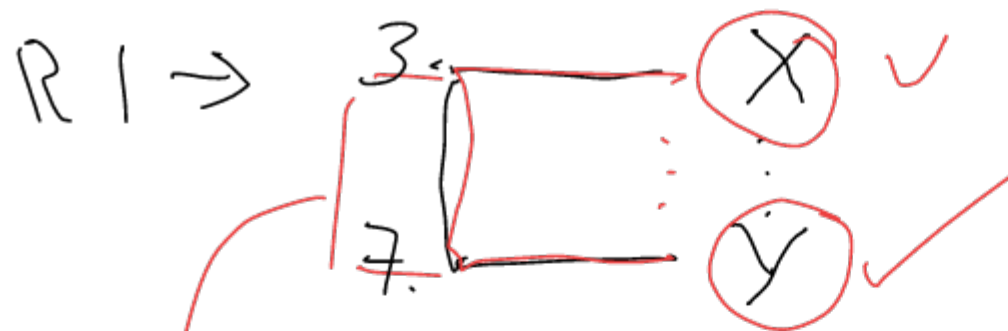
RE → (Modus Ponens - MP)

$$3 - x \rightarrow y$$

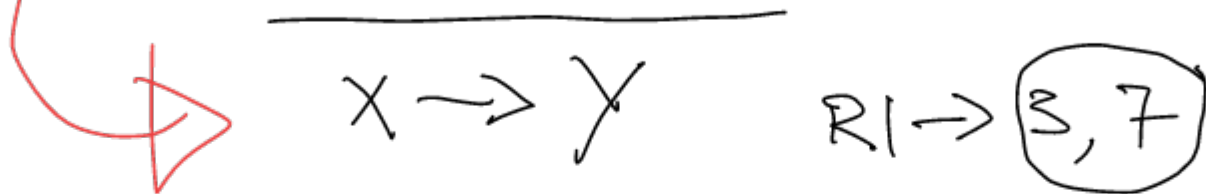
$$4 - x$$

$$\frac{\quad}{y}$$

RE → 3, 4



$X \rightarrow Y$



- 1. $p \rightarrow (q \rightarrow r)$
- 2. p
- 3. $r \rightarrow (s \rightarrow t)$
- 4. $\neg t$

$$\vdash \neg(q \wedge s)$$

$$[DM \quad \neg q \vee \neg s]$$

$$1^\circ - q, s$$

$$2^\circ - RI \neg$$

(R17)

$$5. \quad q \wedge s$$

$$6. \quad q \rightarrow r \quad RE \rightarrow 1, 2$$

$$7. \quad q \quad RE \wedge 5$$

$$8. \quad r \quad RE \rightarrow 6, 7$$

$$9. \quad s \rightarrow t \quad RE \rightarrow 3, 8$$

$$10. \quad s \quad RE \wedge 5$$

$$11. \quad t \quad RE \rightarrow 9, 10$$

$$12. \quad t \wedge \neg t \quad RI \wedge 4, 11$$

$$13. \quad \neg(q \wedge s) \quad RI \neg 5, 12$$

-1. $(p \wedge q)$ $\vdash r$

-2. $(p \rightarrow r)$

3. p RE $\wedge 1$

4. r RE $\rightarrow 2, 3$



$$-1. q \rightarrow \neg(r \rightarrow s)$$

$$-2. (r \rightarrow s) \vee p \leftarrow$$

$$-3. p \rightarrow t$$

$$-4. \neg t$$

$$5. q$$

$$6. \neg(r \rightarrow s) \text{ RE} \Rightarrow 1, 5$$

$$7.$$

$$\vdash \neg q$$

$$x \vee y$$

$$\begin{array}{|l} x \\ \hline \end{array} \neg$$

$$\begin{array}{|l} x \\ \hline \end{array} \neg$$

$$\neg$$

$$1. p \wedge \neg q$$

$$2. p \rightarrow \neg r$$

$$3. \neg (q \vee \neg s)$$

$$4. r \vee s$$

$$5. p \quad \text{RE 1}$$

$$6. \neg r \quad \text{RE} \rightarrow 2, 5$$

$$7.$$

$$\vdash \neg (r \vee s)$$

$$\neg r \wedge \neg s$$

Sil. Disy.
De Morgan

REV

$$\begin{array}{r} \text{XVY} \\ \boxed{\text{X}} \\ \boxed{\text{Z}} \\ \boxed{\text{Y}} \\ \boxed{\text{Z}} \\ \hline \text{Z} \end{array}$$

$$\underline{\text{Z} = \text{X}} \quad \text{ó} \quad \text{Z} = \text{Y}$$

3. r VS

$$\begin{array}{l} \boxed{5} \text{ r} \\ \boxed{17} \text{ S} \end{array}$$

$$\begin{array}{l} \boxed{18} \text{ S} \\ \boxed{20} \text{ S} \end{array}$$

$$\boxed{21} \underline{\underline{\text{S}}}$$

REV

$$1. p \rightarrow q \quad \vdash q$$

$$2. \neg p \rightarrow q$$

$$3. p \vee \neg q$$

$$4. \neg q$$

$$5. \neg p \quad \text{MT } 1, 4$$

$$6. q \quad \text{RE} \rightarrow 2, 5$$

$$7. \neg q \wedge q \quad \text{RI} \wedge 4, 6$$

$$8. \neg \neg q \quad \text{RI} \neg 4-7$$

$$9. \underline{\underline{q}} \quad \text{DN } 8$$

$$1. p \rightarrow \neg \neg q$$

$$2. p \wedge r$$

$$3. (q \vee s) \rightarrow t$$

$$4. p \quad \left. \begin{array}{l} 5. r \end{array} \right\} RE \wedge 2$$

$$6. \neg \neg q \quad RE \rightarrow 1, 4$$

$$7. q \quad EN \geq 6$$

$$8. q \vee s \quad RI \vee 7$$

$$9. t \quad RE \rightarrow 3, 8$$

$$10. t \wedge r \quad RI \wedge 9, 5$$

$$11. \underline{\underline{p \wedge (t \wedge r)}} \quad 4, 10$$

$$\underline{p \wedge (t \wedge r)}$$

Formas lógicas
p. 99

Q

1. $p \leftrightarrow q$
2. $(q \vee r) \leftrightarrow s$
3. $\neg \neg p$
4. $p \quad \text{EN}_2 3$
5. $p \rightarrow q \quad \text{RE} \leftrightarrow 1$
6. $q \quad \text{RE} \rightarrow 5, 4$
7. $q \vee r \quad \text{RI} \vee 6$
8. $(q \vee r) \rightarrow s \quad \text{RE} \leftrightarrow 2$
9. $s \quad \text{RE} \rightarrow 8, 7$
10. $p \wedge s \quad \text{RI} \wedge 4, 9$
11. $(p \wedge s) \vee r \quad \text{RI} \vee 10$

$$\vdash (p \wedge s) \vee r$$

p. 100 (Form. Log.)

$$X \leftrightarrow Y$$

$$\frac{X \leftrightarrow Y}{(X \rightarrow Y) \wedge (Y \rightarrow X)}$$

$$(X \rightarrow Y) \wedge (Y \rightarrow X)$$

$$E \leftrightarrow \begin{cases} X \leftrightarrow Y \\ \hline X \rightarrow Y \end{cases}$$

③

$$1. p \rightarrow (q \rightarrow r)$$

$$\vdash (s \wedge \neg r) \vee t$$

$$2. \neg(p \wedge q) \rightarrow s$$

$$3. \neg r$$

$$4. p \wedge q$$

S / objetivo: $\neg(p \wedge q)$

$$5. \phi$$

} RE $\wedge$ 4

$$6. q$$

$$7. q \rightarrow r \quad RE \rightarrow 1, 5$$

$$8. r \quad RE \rightarrow 7, 6$$

$$9. r \wedge \neg r \quad RI \wedge 8, 3$$

$$10. \neg(p \wedge q) \quad RI \neg 4-9$$

$$11. s \quad RE \rightarrow 2, 10$$

$$12. s \wedge \neg r \quad RI \wedge 11, 3$$

$$13. \underline{(s \wedge \neg r) \vee t} \quad RI \vee 12$$

$$\text{CP: } \frac{X}{Y \rightarrow X} \quad \left| \frac{\frac{Y \rightarrow X}{\neg Y \vee X}}{\quad} \right.$$

$$\vdash p \Rightarrow q$$

$$\vdots$$

$$\neg 3. q$$

$$4. \underline{\underline{p \Rightarrow q}} \text{ CP3}$$

$$\text{ECQ}_2: \frac{X}{\neg X \Rightarrow Y}$$

$$\begin{array}{c} X \\ \hline X \vee Y \end{array}$$

$$\frac{\neg X}{X \Rightarrow Y}$$

$$\frac{\neg X}{\neg X \vee Y}$$

$$1. \neg(p \vee \neg q)$$

$$2. (\neg p \vee q) \rightarrow r$$

$$3. S$$

$$4. \neg(\neg r \vee \neg s) \rightarrow t$$

$$5. \neg p \wedge q \quad DM 1$$

$$6. \neg p \quad RE 5$$

$$7. \neg p \vee q \quad R \vee 6$$

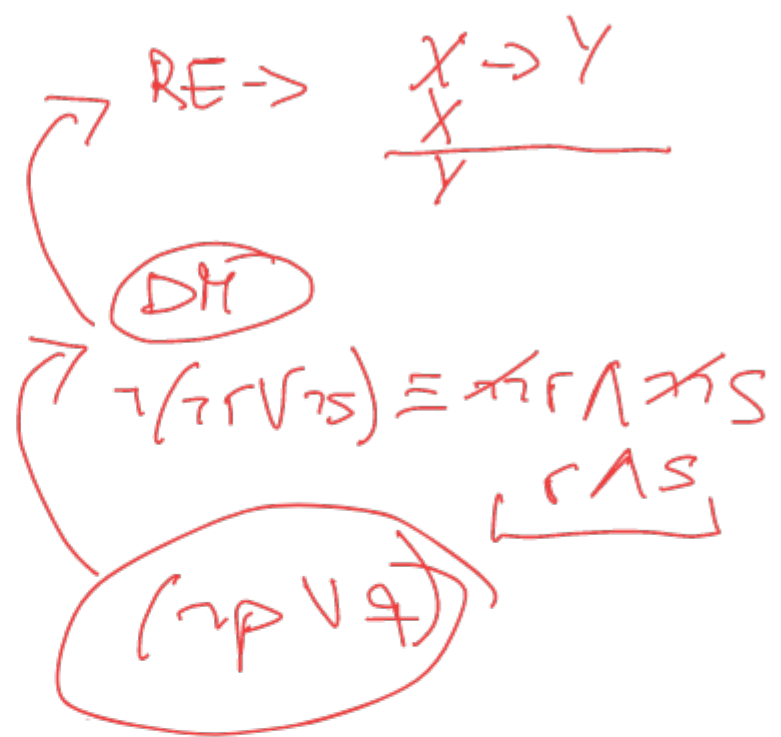
$$8. r \quad RE \rightarrow 2, 7$$

$$9. r \wedge s \quad R \wedge 8, 3$$

$$10. \neg(\neg r \vee \neg s) \quad DM 9$$

$$11. \underline{\underline{t}} \quad RE \rightarrow 4, 10$$

$$\vdash t$$



1. $(p \vee q) \rightarrow r$
 2. $r \rightarrow (s \vee t)$
 3. $t \rightarrow u$
 4. $\neg s \wedge \neg u$
 5. $\neg s$
 6. $\neg u$
 7. $\neg t$
 8. $\neg s \wedge \neg t$
 9. $\neg(s \vee t)$
 10. $\neg r$
 11. $\neg(p \vee q)$
 12. $\neg p \wedge \neg q$
 13. $\neg q$
- REA 4
 MT 3, 6
 RIA 5, 7
 DM 8
 MT 2, 9
 MT 1, 10
 DM 11
 REA 12

$$\neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q \quad \text{DM}$$

$$\frac{\begin{array}{c} X \rightarrow Y \\ \neg Y \\ \hline \neg X \end{array}}{\neg(s \vee t)} \quad \text{MT}$$

$$\frac{\neg(s \vee t)}{\neg r} \quad \text{DM}$$

$$\frac{\neg r}{\neg s \wedge \neg t} \quad \text{MT}$$

$$\boxed{\neg t}$$

$$1. p \wedge (\neg q \vee r)$$

$$2. ((p \wedge q) \vee (p \wedge r)) \rightarrow s$$

$$3. (s \vee t) \rightarrow \neg(u \wedge \neg r)$$

$$4. (p \wedge q) \vee (p \wedge r) \text{ Distr. C1}$$

$$5. s \quad RE \rightarrow 2, 4$$

$$6. s \vee t \quad R1V \ 5$$

$$7. \neg(u \wedge \neg r) \quad RE \rightarrow 3, 6$$

$$8. \underline{\underline{u \rightarrow r}} \quad IC \rightarrow, \wedge 7$$

$$\vdash u \rightarrow r$$

$$\equiv \neg(u \wedge \neg r)$$

$$(s \vee t)$$

$$s$$

- 1. $p \vee \neg p$
- 2. $(p \rightarrow q) \wedge (q \rightarrow r)$
- 3. $(\neg p \rightarrow s) \wedge (s \rightarrow r)$

- 4. p
- 5. $p \rightarrow q$ $E \wedge 2$
- 6. q $E \rightarrow 5, 4$
- 7. $q \rightarrow r$ $E \wedge 2$
- 8. r $E \rightarrow 7, 6$

- 9. $\neg p$
- 10. $\neg p \rightarrow s$ $E \wedge 3$
- 11. s $E \rightarrow 10, 9$

$\vdash r$

- 12. $s \rightarrow r$ $E \wedge 3$
- 13. r $E \rightarrow 12, 11$

14. r $E \vee 1, 4-8, 9-13$

$R \vee$

$X \vee Y$

$\begin{bmatrix} X \\ Z \end{bmatrix}$

$\begin{bmatrix} Y \\ Z \end{bmatrix}$

Z

$$-1. p \rightarrow (q \rightarrow r)$$

$$-2. p$$

$$-3. r \rightarrow (s \rightarrow t)$$

$$-4. \neg t$$

$$\vdash \neg(q \wedge s)$$

$$\neg q \vee \neg s$$

$$5. q \wedge s$$

$$6. q \rightarrow r \quad E \rightarrow 1, 2$$

$$7. q \quad E \wedge 5$$

$$8. r \quad E \rightarrow 6, 7$$

$$9. s \rightarrow t \quad E \rightarrow 3, 8$$

$$10. \neg s \quad MT 9, 4$$

$$11. s \quad E \wedge 5$$

$$12. \neg s \wedge s \quad I \wedge 10, 11$$

$$13. \neg(q \wedge s) \quad RI \neg 5-12$$

$$\frac{x \rightarrow y}{\neg(x \wedge \neg y)} \quad [\neg x \vee y]$$

$$MT \quad \frac{x \rightarrow y \quad \neg y}{\neg x}$$

$$RI \neg \quad \frac{\neg x \quad [y \wedge \neg y]}{\neg x}$$

- 1. $p \rightarrow q$
 -2. $q \rightarrow r$
 -3. $r \rightarrow \neg s$
 -4. $(p \rightarrow \neg s) \rightarrow (q \rightarrow p)$
 -5. $\neg p$
 6. $p \rightarrow r$ Tr. Cond. 1, 2
 7. $q \rightarrow \neg s$ 1, 2, 3
 8. $p \rightarrow \neg s$ 1, 6, 3
 9. $q \rightarrow p$ $\rightarrow$ 4, 8
 10. $\neg q$ MT 9, 5
 =

$\vdash \neg q$

$$X \leftrightarrow Y$$

$$\underline{\underline{}}$$

$$(X \rightarrow Y) \wedge (Y \rightarrow X)$$

/

/

$$\neg X \vee Y$$

1