Matemáticas II.

Clasificación de Cuádricas y Formas reducidas

Notación:

$Q \equiv XAX^T = 0, \quad A = A^T, \quad |A|,$

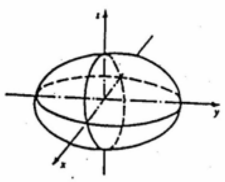
• Invariantes Euclídeos: $|A|$, A_{00}, valores propios de (A_{00}), $tr((A_{00}))$, J.

$$A_{00} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} \qquad (A_{00}) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \qquad \mathbf{J} = \begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{13} \\ a_{13} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{22} & a_{23} \\ a_{23} & a_{33} \end{vmatrix}, \qquad \mathbf{tr}((\mathbf{A_{00}})) = a_{11} + a_{22} + a_{33}.$$

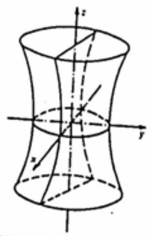
• Ecuación característica de (A_{00}): $|(A_{00}) - \lambda I_3| = (-1)^3\lambda^3 + (-1)^2 tr((A_{00}))\lambda^2 + (-1)J\lambda + A_{00} = 0$

Cuádricas Regulares (con centro)

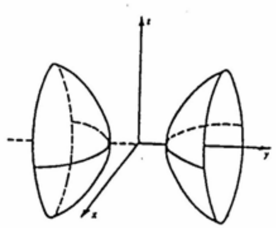
<table>
<tr><th>$|A|$</th><th>A_{00}</th><th>Forma reducida</th><th>d_{ii}</th><th>sig(d_{ii}) = signo de d_{ii}</th><th>Clasificación</th></tr>
<tr><td rowspan="4">$|A| \neq 0$</td><td rowspan="2">$A_{00} \neq 0$</td><td rowspan="2">$X \begin{pmatrix} d_{00} & 0 & 0 & 0 \\ 0 & d_{11} & 0 & 0 \\ 0 & 0 & d_{22} & 0 \\ 0 & 0 & 0 & d_{33} \end{pmatrix} X^T = 0$
$d_{00}x_0^2 + d_{11}x_1^2 + d_{22}x_2^2 + d_{33}x_3^2 = 0$</td><td rowspan="2">$d_{11}, d_{22}, d_{33}$ son las raices de la ecuación característica de (A_{00}).
$d_{00} = \frac{|A|}{A_{00}}$</td><td>sig($d_{11}$) =sig($d_{22}$) =sig($d_{33}$).
Elipsoide.</td><td>$|A| > 0 \Rightarrow$ Elipsoide Imaginario
$|A| < 0 \Rightarrow \begin{cases} \text{Elipsoide Real} \\ \text{Esfera si} \\ a_{11} = a_{22} = a_{33} \text{ y} \\ a_{12} = a_{13} = a_{23} = 0 \end{cases}$</td></tr>
<tr><td>sig(d_{11}) =sig(d_{22}) $\neq$sig(d_{33}).
Hiperboloide</td><td>$|A| > 0 \Rightarrow$ Hiperboloide Reglado
$|A| < 0 \Rightarrow \begin{cases} \text{Hiperboloide no Reglado} \\ \text{o de dos hojas.} \end{cases}$</td></tr>
<tr><td rowspan="2">$A_{00} = 0$</td><td rowspan="2">$X \begin{pmatrix} 0 & 0 & 0 & d_{03} \\ 0 & d_{11} & 0 & 0 \\ 0 & 0 & d_{22} & 0 \\ d_{03} & 0 & 0 & 0 \end{pmatrix} X^T = 0$
$d_{11}x_1^2 + d_{22}x_2^2 + 2d_{03}x_0x_3 = 0$</td><td rowspan="2">$d_{11}, d_{22}, 0$ son las raices de la ecuación característica de (A_{00}).
$d_{03} = \pm\sqrt{\frac{|A|}{J}}$</td><td>sig($d_{11}$) =sig($d_{22}$).</td><td>$|A| < 0 \Rightarrow$ Paraboloide no Reglado (Elíptico)</td></tr>
<tr><td>sig(d_{11}) $\neq$sig(d_{22})</td><td>$|A| > 0 \Rightarrow$ Paraboloide Reglado (Hiperbólico)</td></tr>
</table>



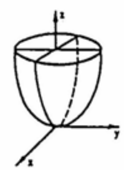
ELIPSOIDE



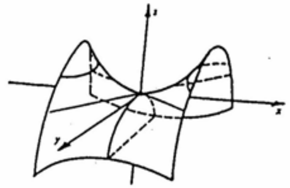
HIPERBOLOIDE REGLADO O DE UNA HOJA



HIPERBOLOIDE DE DOS HOJAS



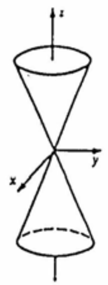
PARABOLOIDE ELIPTICO



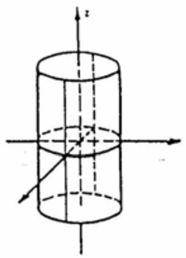
PARABOLOIDE HIPERBÓLICO

Cuádricas Degeneradas

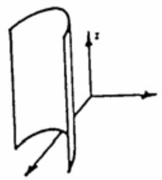
$\lvert A\rvert$	rango de A	Puntos Singulares $SA=(0,0,0,0)$	Centros	rango de (A_{00})	Clasificación
$\lvert A\rvert=0$	$\text{rg}(A)=3$	S punto singular propio único	un único centro $Z=S$	$\text{rg}((A_{00}))=3$	$\text{sigt}(Q)=1 \Rightarrow$ Cono Real $\text{sigt}(Q)=3 \Rightarrow$ Cono Imaginario
		S_∞ punto singular impropio	$\exists r_Z$ recta de centros propia $S_{infty}\in r_Z$	$\text{rg}((A_{00}))=2$	$\text{sig}(\lambda_1)\neq\text{sig}(\lambda_2)$ Cilindro Hiperbólico $\text{sig}(\lambda_1)=\text{sig}(\lambda_2)$ Cilindro Elíptico
			$\exists r_Z$ recta impropia de centros	$\text{rg}((A_{00}))=1$	Cilindro parabólico
	$\text{rg}(A)=2$	$\exists r_S$ recta propia de puntos singulares	$\exists r_Z=r_S$	$\text{rg}((A_{00}))=2$	C_∞ par de rectas reales que se cortan $\Rightarrow$ Par de Planos reales que se cortan $(\text{sig}(\lambda_1)\neq sig(\lambda_2))$
					C_∞ par de rectas imag. que se cortan $\Rightarrow$ Par de Planos imag. que se cortan $(\text{sig}(\lambda_1)=\text{sig}(\lambda_2))$
		$\exists r_S$ recta impropia	$\exists\Pi_Z$ plano propio de centros de recta impropia r_S	$\text{rg}((A_{00}))=1$	Par de planos paralelos (reales, si $Q\cap r$, con $r\neq r_S$, es de puntos reales) (imaginarios si dicha intersección es imaginaria).
	$\text{rg}(A)=1$				Plano doble



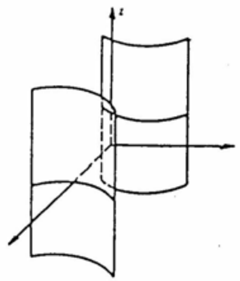
CONO ELIPTICO



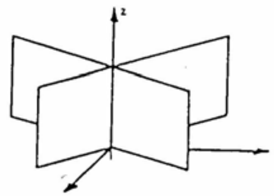
CILINDRO ELIPTICO



CILINDRO PARABÓLICO



CILINDRO HIPERBÓLICO



DOS PLANOS QUE SE CORTAN